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Inflationary Phase in a Generalized Brans-Dicke Theory

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Abstract We find a solution for exponential inflation in a Brans-Dicke generalized model, where the coupling "constant" is variable. While in General Relativity the equation of state is $p = -\rho$, here we find $p = \alpha\rho$, where $\alpha < -2/3$. The negativity of cosmic pressure implies acceleration of the expansion, even with $\Lambda < 0$.

Keywords Inflation · Brans-Dicke · Scalar-tensor · Cosmology · Coupling "constant" · Cosmological "constant"

1 Introduction

New evidence for primordial inflation has been recently gathered through cosmic microwave observation [9]. Barrow [1] has pointed out the possible relevance of scalar-tensor gravity theories in the study of the inflationary phase during the early Universe. He obtained exact solutions for homogeneous and isotropic cosmologies in vacuum and radiation cases, for a variable coupling "constant ", $\omega = \omega(\phi)$, where ϕ stands for the scalar field. For accounts on inflation, see, for instance, Linde's book [2].

In this letter we extend Barrow's paper by the study of an inflationary exponential phase. This letter can be considered also as a complement to Berman and Som's paper [3] dealing with the inflationary phase in B.D. original framework, which was followed by a letter by Berman [4] where he studied the same problem in the context of a B.D. theory endowed with a cosmological constant. For scalar-tensor theories, consult the books by Berman [10], Faraoni [11], and Fujii and Maeda [12]. In Berman [13], we find a *rationale* for the existence of a cosmological "constant", though we must remember that a negative cosmic pressure may be also responsibilized for accelerated expansion, which includes exponential inflation.

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2 The Field Equations

One way to formulate a scalar-tensor theory of gravitation can be with the following Lagrangian:

$$L_\phi = -\phi R + \phi^{-1}\omega(\phi)\partial_a\phi\partial^a\phi + 16\pi L_m - 2\Lambda(\phi) \tag{1}$$

where L_m is the Lagrangian for matter fields, and ϕ is the scalar field. If $\omega = \text{const}$ we obtain the Brans-Dicke [5] theory. This Lagrangian was adopted by Barrow and Maeda [6]. For a discussion about the Lagrangians of the scalar theories of gravitation, see [7]. The cosmological term $\Lambda(\phi)$ is taken also to mean time-dependent lambda.

By varying the action associated with (1) with respect to the space-time metric and the scalar field ϕ, respectively we obtain the generalized Einstein equations and the wave equation for ϕ [1]:

$$G_{ab} = -\frac{8\pi}{\phi}T_{ab} - \frac{\omega}{\phi^2}\left[\phi_a\phi_b - \frac{1}{2}g_{ab}\phi_i\phi^i\right] - \frac{1}{\phi}\left[\phi_{a;b} - g_{ab}\Box\phi\right] - \frac{\Lambda}{\phi}g_{ab} \tag{2}$$

$$[3+2\omega]\Box\phi = 8\pi T - \left(\frac{d\omega}{d\phi}\right)\phi_i\phi^i + 2\phi\frac{d\Lambda}{d\phi} - 4\Lambda \tag{3}$$

In General Relativity theory, in face of a perfect fluid matter field, from the field equations, it is derived the energy momentum conservation law,

$$T^{ab}_{;b} = 0 \tag{4}$$

In Brans-Dicke theory, Weinberg [14] has commented that in order to preserve the Principle of Equivalence, the scalar-field does not enter into the conservation equation above, which takes into consideration only the matter-fields. For scalar-tensor theories, as well, this conservation equation is imposed on the same token, but, of course, if we take the field equations, say, for Robertson-Walker's metric, obtaining an equation for cosmic pressure and other for the energy density, we could combine those equations, along with the scalar-field one, and obtain a generalisation of the kind,

$$G^{ab}_{;b} = 0$$

where the conservation law applies to the right-hand-side of (2).

With Robertson-Walker's metric,

$$ds^2 = dt^2 - a^2\left[\left(1-kr^2\right)^{-1}dr^2 + r^2d\theta + r^2\sin^2\theta d\varphi^2\right] \tag{5}$$

we find, from (4), (2) and (3), that :

$$\frac{\dot a^2}{a^2} + \frac{k}{a^2} = \frac{8\pi\rho}{3\phi} - \frac{\dot\phi}{\phi}\frac{\dot a}{a} + \frac{\omega}{6}\frac{\dot\phi^2}{\phi^2} + \frac{\Lambda}{3\phi} \tag{6}$$

$$\dot\rho + 3\frac{\dot a}{a}(\rho + p) = 0 \tag{7}$$

$$\ddot\phi + \left[3\frac{\dot a}{a} + \frac{\dot\omega}{2\omega+3}\right]\dot\phi = \frac{1}{3+2\omega}\left[8\pi(\rho - 3p) - 2\phi\frac{\dot\Lambda}{\dot\phi} + 4\Lambda\right] \tag{8}$$

where overdots stand for time derivatives.

From now on, we consider only spatially flat solutions ($k = 0$). Let

$$a = a_0 e^{Ht} \tag{9}$$

where a_0, H, are constants, and

$$p = \alpha\rho \tag{10}$$

($\alpha = \text{const}$).

In General Relativity, $\alpha = -1$ for the inflationary phase; here, we must consider also other possibilities. Law (10) stands for a "perfect" gas equation of state. From (7) and (9), we find, using (10),

$$\rho = \rho_0 e^{-3H(1+\alpha)t} \tag{11}$$

where $\rho_0 = \text{const}$.

Remember that $H = \dot{a}/a$ stands for Hubble's parameter.

Consider the solution for $\phi(t)$, and $\omega(t)$:

$$\frac{\dot{a}^2}{a^2} = -\frac{\dot{\phi}}{\phi}\frac{\dot{a}}{a} \tag{12}$$

and,

$$\frac{8\pi\rho}{\phi} = -\frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2} - \frac{\Lambda}{3\phi} \tag{13}$$

By summing (12) and (13), we recover expression (6), so that the above two equations, result in a particular solution, though that it has some generality altogether.

We find from (12),

$$\phi(t) = \phi_0 e^{-Ht} \tag{14}$$

with $\phi_0 = \text{const}$.

From (13), we get a possible solution with, $\omega \gg 3/2$,

$$\omega \cong \omega_0 e^{-H(2+3\alpha)t} \tag{15}$$

with $\omega_0 = \text{positive constant}$, and $\phi_0 > 0$.

The constants must obey the condition, obtained from (12) and (13),

$$8\pi\rho_0 + \frac{1}{3}\Lambda_0 + \frac{1}{6}H^2\phi_0\omega_0 = 0$$

The reason for a positive scalar-field, is that gravitation should be kept attractive, i.e., Newton's gravitational constant is positive. The reason for a positive coupling "constant", is that experimental gravitational and astronomical observations require a large positive ω value.

It is then, highly desirable that ω grow with time, so we impose,

$$\alpha < -\frac{2}{3} \tag{16}$$

This condition on the equation of state encompasses the case $\alpha = -1$ of G.R.

From the scalar-field equation, of course, we get the fulfilled approximate condition,

$$2H^2\phi_0 \cong \frac{M}{(3+2\omega)} e^{-H(2+3\alpha)t} \tag{17}$$

where,

$$M = \omega_0 H^2 \phi_0^2 (2+3\alpha) + 4\Lambda_0 + 6(1+\alpha)\Lambda_0 - 8\pi\rho_0 (1-3\alpha) \tag{18}$$

3 Conclusion

We have thus, found a new solution for inflation, that deserves attention. On the other hand, it can be shown [8] that condition (16) is necessary for the amplification of gravitational waves during exponential inflation, at least when ω is constant.

It could be argued that another possible solution would be given by a positive cosmological "constant", followed by a negative coupling ω. It certainly may be more palatable for string theorists, but it would require some hand waving of the type $\omega_0 < -2$, because otherwise, the observation of slowly moving particles or of time-dilation experiments, which imply (see [14]),

$$G = \left[\frac{2\omega+4}{2\omega+3}\right]\phi^{-1} \tag{19}$$

would again carry $G < 0$.

We found,

$$\Lambda = \Lambda_0 e^{-3H(1+\alpha)t} \tag{20}$$

where $\Lambda_0 = \text{constant}$.

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